

Modelling trend progression through an extension of the Polya Urn Process

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Abstract. Knowing how and when trends are formed is a frequently visited research goal. In our work, we focus on the progression of trends through (social) networks. We use a random graph (RG) model to mimic the progression of a trend through the network. The context of the trend is not included in our model. We show that every state of the RG model maps to a state of the Polya process. We find that the limit of the component size distribution of the RG model shows power-law behaviour. These results are also supported by simulations.

Keywords: Retweet graph, Twitter, graph dynamics, random graph model, Polya process

1 Introduction

How can we reach a large audience with our message? What is the best way to reach a large audience with an advertisement? These are questions that are asked many times in our modern day society. Not only for corporate interest, but also for public interest by governments and charities. Everyone wants to get their message across to a large audience. Finding out how to do this is a frequently visited research goal in many fields, e.g. economics [23], evolutionary biology [27,8] and physics [20].

In our work, we focus on the progression of trends through the network of users. Incident to our approach, we focus on the microscopic dynamics of user-to-user interaction to derive the overall behaviour, which is similar to the approach used in other works, e.g. [25,11]. Our work differs from these in that we model the spread of the messages in a step-by-step fashion, whereas [25,11] use a given degree distribution per user as a start of their analysis.

Our goal is to devise a model that mimics the progression of a trend through (social) networks. By doing this, we focus only on the pattern of progression of the trends and not their content. Based on observations from *Twitter* data, we have built a model that captures the different changes that occur in a network whilst a topic is spreading. In [24], we derived basic growth properties of the model and the speed of convergence of these properties. In this paper, we derive the component size distribution of the model.

In Section 2, we describe related fields of study, and in Section 3 we introduce the RG model and the Polya process. Here we show that the RG model can be easily mapped to the Polya process. Then, in Section 4, we derive the behaviour of the component size distribution. Finally, we state our conclusions and discuss the possibilities for further research in Section 5.

2 Related work

There are many studies that focus on information diffusion in online social networks. In [12], Guille et al. provide a survey, in which they distinguish two approaches: a descriptive approach and a predictive approach. A few examples of these approaches are given below.

A large number of descriptive studies into information diffusion have added to the knowledge about how messages progress through online social networks. Lerman and Ghosh [17] find that diffusion of information in *Twitter* and *Digg* is very dependent on the network structure. Bhattacharya and Ram [4] study the diffusion of news items in *Twitter* for several well-known news media and find that these cascades follow a star-like structure. Friggeri et al. [10] study rumors in Facebook and find bursty spreading patterns. Sadikov and Martinez [22] find that on *Twitter*, tags tend to travel to distant parts of the network and URLs tend to travel shorter distances. Bhamidi et al. [3] propose and validate a so-called superstar random graph model for a giant component of a retweet graph. Hoang and Lim [13] propose a dynamic model that uses both user and item virality and user susceptibility as descriptors to model information diffusion. Iwata et al. [14] use an inhomogeneous Poisson Process to model the diffusion of item adoption. Another angle to model information diffusion uses epidemic spreading: By a maximum entropy argument Bauckhage et al. [2] find a closed form expression for the path length distribution in a network. Finally, Carton et al. [5] propose to perform an audience analysis when analysing diffusion processes, thus including not only the diffusers in the analysis but also the receivers of the content.

Romero et al. [21] use the predictive approach to analyse the spread mechanics of content through hashtag use and derive probabilities that users adopt a hashtag. Kupavskii et al. [15] predict the number of retweets based on several features. They find that the flow of a message is one of the most important features in the prediction. Altshuler et al. [1] use past information and diffusion models to derive a lower bound on the probability that a topic becomes trending. Zaman et al. [28] predict future retweets based on features at the user level. Wu and Raschid [26] define a user specific potential function which reflects the likelihood of a follower sharing the users content in the future.

Classification and clustering of trends on *Twitter* has also attracted considerable attention in the literature. Zubiaga et al. [29] derive four different types of trends, using fifteen features to make their distinction. They distinguish trends triggered by news, current events, memes or commemorative tweets. Lehmann et al. [16] study different patterns of hashtag trends in *Twitter*. They also observe four different classes of hashtag trends. Rattanaritnont et al. [19] propose

to distinguish topics based on four factors, namely cascade ratio, tweet ratio, time of tweet and patterns in topic-sensitive hashtags. Ferrara et al. [9] cluster memes based on content and user, tweet and network similarity measures.

We use the analysis of urn processes in this paper, in contrast to other works in this area. Pemantle [18] presents a survey of different techniques that are used in this field of research. In this work, we focus on extensions of the Polya urn problem, which is thoroughly analysed by Chung et al. in [6]. Specifically, we are interested in the infinite generalized Polya urn model, as studied in Section 4 of [6].

3 Problem formulation

In this section, we first describe the setup of the RG model. Then, the Polya process is introduced. Finally, we show that every state of the model maps to a state of the Polya process.

Our main object of study is the retweet graph $G = (V, E)$, which is a graph of users that have participated in the discussion of a specific topic. A directed edge $e = (u, v)$ indicates that user v has retweeted a tweet of u . We observe the retweet graph at the time instances $t = 0, 1, 2, \dots$, where either a new node or a new edge is added to the graph, and we denote by $G_t = (V_t, E_t)$ the retweet graph at time t . As usual, the out- (in-) degree of node u is the number of directed edges with source (destination) in u . For every new message initiated by a new user u a tree H_u is formed. Then, \mathcal{T}_t denotes the forest of message trees. Note that in the RG model, a new message from an already existing user u (that is, $u \in \mathcal{T}_t$) does not initiate a new message tree. We define $|\mathcal{T}_t|$ as the number of new users that have started a message tree up to time t . I.e. G_t can be seen as an simple representation of the union of message trees $\cup_{H_u \in \mathcal{T}_t} H_u$.

The goal of the model is to capture the development of trending behaviour. We do this by combining the spread of several messages. As a result of this approach, we first need to model the progression of a single message in the network. To this end, we use the superstar model of Bhamidi et al. [3] for modelling distinct components of the retweet graph, to which we add the mechanism for new components to arrive and the existing components to merge. In this paper, our aim is to analyse the component size distribution of G_t . For the sake of simplicity of the model, we neglect the friend-follower network of *Twitter*. Note that in *Twitter* every user can retweet any message sent by any public user, which supports our simplification.

We consider the evolution of the retweet graph in time $(G_t)_{t \geq 0}$. We use a subscript t to indicate G_t and related notions at time t . We omit the index t when referring to the graph at $t \rightarrow \infty$. Let G_0 denote the graph at the start of the progression. In the analysis of this paper, we assume G_0 consists of a single node. Note that in reality, this does not need to be the case: any directed graph can be used as an input graph G_0 .

Recall that G_t is a graph of *users*, and an edge (u, v) means that v has retweeted a tweet of u . We consider time instances $t = 1, 2, \dots$ when either a

new node or a new edge is added to the graph G_{t-1} . We distinguish three types of changes in the retweet graph:

- T_1 : a new user u has posted a new message on the topic, so node u is added to G_{t-1} ;
- T_2 : a new user v has retweeted an existing user u , so node v and edge (u, v) are added to G_{t-1} ;
- T_3 : an existing user v has retweeted another existing user u , so edge (u, v) is added to G_{t-1} .

Note that the initial node in G_0 is equivalent to a T_1 arrival at time $t = 0$. Assume that each change in G_t at $t = 1, 2, \dots$ is T_1 with probability $\lambda/(1 + \lambda)$, independently of the past. Also, assume that a new edge (retweet) is coming from a new user with probability p . Then, the probabilities of T_1 , T_2 and T_3 arrivals are, $\frac{\lambda}{\lambda+1}$, $\frac{p}{\lambda+1}$, $\frac{1-p}{\lambda+1}$ respectively. The parameter p governs the process of components merging together, while λ governs the arrival of new components in the graph.

For both T_2 and T_3 arrivals we define the same mechanism to choose the source of the new edge (u, v) as follows.

Let u_0, u_1, \dots be the users that have been added to the graph as T_1 arrivals, where u_0 is the initial node. Denote by $H_{i,t}$ the subgraph of G_t that includes u_i and all users that have retweeted the message of u_i in the interval $(0, t]$. We call such a subgraph a message tree with root u_i . We assume that the probability that a T_2 or T_3 arrival at time t will attach an edge to one of the nodes in $H_{i,t-1}$ with probability $p_{H_{i,t-1}}$ is proportional to the size of the message tree:

$$p_{H_{i,t-1}} = \frac{|H_{i,t-1}|}{\sum_{H_{j,t-1} \subset \mathcal{T}_{t-1}} |H_{j,t-1}|}.$$

This creates a preferential attachment mechanism in the formation of the message trees.

For the selection of the source node, we use the superstar model, with parameter q chosen uniformly for all message trees. This model was suggested in [3] for modelling the largest connected component of the retweet graph on a given topic, in order to describe a progression mechanism for a single retweet tree. Our extensions compared to [3] are that we allow new message trees to appear (T_1 arrivals), and that different message trees may either remain disconnected or get connected by a T_3 arrival.

For a T_3 arrival, the target of the new edge (u, v) is chosen uniformly at random from V_{t-1} , with the exception of the earlier chosen source node u , to prevent self-loops. That is, any user is equally likely to retweet a message from another existing user. Thus, after a T_3 arrival a message tree can have cycles.

Note that we do not include tweets and retweets that do not result in new nodes or edges in the retweet graph. This could be done, for example, by introducing dynamic weights of vertices and edges, that increase with new tweets and retweets. Here, we consider only an unweighted model.

Polya process

In our analysis of the previously stated model, we use the Polya process, which is defined in [6] as follows:

Given two parameters $\gamma \in \mathbb{R}, 0 \leq \bar{p} \leq 1$, we start with one bin, containing one ball. We then introduce balls one at a time. For each new ball, with probability \bar{p} , we create a new bin and place the ball in that bin; with probability $1 - \bar{p}$, we place the ball in an existing bin (of size m), such that the probability that the ball is placed in a bin is proportional to m^γ .

We only consider the case where $\gamma = 1$ in this paper.

Let $f_{i,t}$ denote the fraction of bins that contain i balls at time t . In [6], the authors find that under the following assumptions:

- i) for each i , there exists $f_i \in \mathbb{R}^+$ s.t. a.s. $\lim_{t \rightarrow \infty} f_{i,t}$ exists and is equal to f_i ,
- ii) a.s. $\lim_{t \rightarrow \infty} \sum_{j=1}^{\infty} f_{j,t} j^\gamma$ exists, is finite, and is equal to $\sum_{j=1}^{\infty} f_j j^\gamma$.

The limit of the fraction of bins that contain i balls (denoted by f_i) satisfies

$$f_i \propto i^{-(1+1/(1-\bar{p}))}.$$

Mapping from retweet graph to balls and bins

In this section, we show that every retweet graph G_t can be mapped to a state of the Polya process, with $\gamma = 1$ and $\bar{p} = \frac{\lambda}{\lambda+1}$.

Lemma 1 *Every retweet graph G_t can be represented as a state S of the Polya process.*

Proof. Suppose we have a retweet graph G_t , that consists of k components of known sizes, moreover $G_t = \{C_1, C_2, \dots, C_k \mid |C_1|, |C_2|, \dots, |C_k|\}$. For instance, in Figure 1a $G_t = \{C_{green}, C_{yellow}, C_{blue}, C_{red} \mid |C_{green}| = 5, |C_{yellow}| = 2, |C_{blue}| = 4, |C_{red}| = 2\}$. First, we take C_{green} that consists of five nodes and fill a bin with five green balls. Then, we take C_{yellow} , and fill a bin with $|C_{yellow}| = 2$ yellow balls. Next, we take C_{blue} and fill a bin with $|C_{blue}| = 4$ blue balls. Finally, we take C_{red} and fill a bin with $|C_{red}| = 2$ red balls. These four bins with their corresponding balls then form a state S of the Polya process, depicted in Figure 1b. Note that by using this procedure for an arbitrary graph G_t , we can always construct a state S .

A special case: $p = 1$

In this subsection, we show that the RG model with parameter $p = 1$ is equivalent to the Polya process w.p. $\gamma = 1, \bar{p} = \frac{\lambda}{\lambda+1}$. We use this to find the limiting distribution of the component sizes of the RG model.

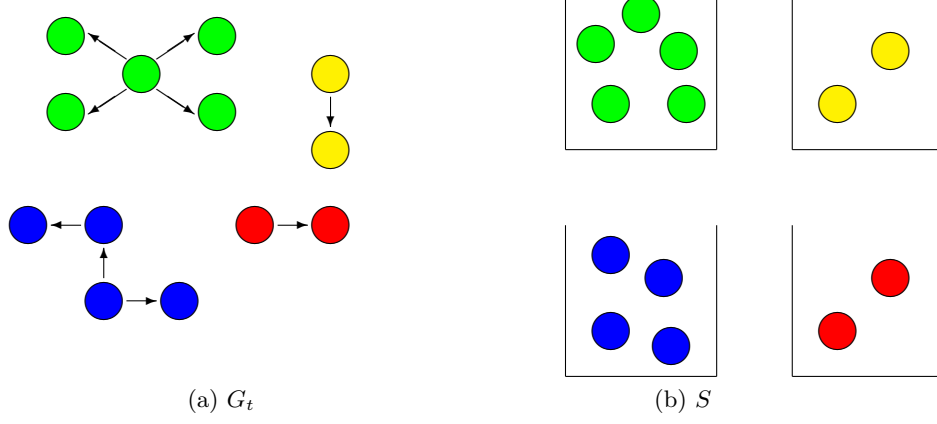


Fig. 1. Mapping from retweet graph G_t (a) to Polya process state S (b).

Theorem 2 *RG model w.p. $p = 1$ is equivalent to a Polya process w.p. $\gamma = 1, \bar{p} = \frac{\lambda}{\lambda+1}$*

Proof. From Lemma 1 we know that a retweet graph G_t can be mapped to a state S of the Polya process. Next we show that the probability distribution of an arrival to G_t is identical to the probability distribution of an addition of a ball to the state S in the Polya process, given $p = 1$ in the RG model.

Since $p = 1$, we have two types of arrivals, T_1 and T_2 . First, we consider a T_1 arrival

$$\mathbb{P}(T_1 \text{ arrival}) = \frac{\lambda}{\lambda+1} = \bar{p} = \mathbb{P}(\text{new bin is created}). \quad (1)$$

Then, for a T_2 arrival, a new node is added to an existing component. The probability that a new node arrives to component i in G_t is

$$\mathbb{P}(\text{arrival to component } i \text{ in } G_t) = \frac{|C_i|}{|V_t|}. \quad (2)$$

For the Polya process, the probability that a new ball arrives in bin i in S is as follows

$$\mathbb{P}(\text{arrival to bin } i \text{ in } S) = \frac{X_i^\gamma}{\sum_{j=1}^k X_j^\gamma} = \frac{X_i}{\sum_{j=1}^k X_j}. \quad (3)$$

Using these equations and Lemma 1, we find that

$$\begin{aligned} \mathbb{P}(\text{arrival to component } i \text{ in } G_t) &= \frac{|C_i|}{|V_t|} = \frac{|C_i|}{\sum_{j=1}^k |C_j|} = \frac{X_i}{\sum_{j=1}^k X_j}, \\ &= \mathbb{P}(\text{arrival to bin } i \text{ in } S). \end{aligned}$$

Thus, the probability distribution of an arrival to the RG model w.p. $p = 1$ is identical to the probability distribution of the Polya process w.p. $\gamma = 1, \bar{p} = \frac{\lambda}{\lambda+1}$.

In combination with Lemma 1, we conclude that the RG model is equivalent to a Polya process with parameters $\gamma = 1$ and $\bar{p} = \frac{\lambda}{\lambda+1}$.

From this equivalence, we immediately obtain the limiting component size distribution of the RG model from [6], given $p = 1$.

Corollary 3 *For f_i , the fraction of components of size i , it holds that*

$$f_i \propto i^{-(\lambda+2)},$$

for the RG model w.p. $p = 1$.

4 Component size distribution

Using the fact that the RG model can be mapped to a Polya process, we derive the limiting behaviour of the component size distribution for general p .

Theorem 4 *For the RG model, the limit f_i of the fraction of components of size i a.s. satisfies*

$$f_i \propto i^{-(1+\frac{\lambda+1}{p})}.$$

Proof. Given the mapping from the retweet graph to the Polya process, we can derive f_i for the RG model similar to the derivation in [6]. In that paper, the authors define $p_{i,t}$ as follows

$$p_{i,t} := \mathbb{P}(\text{ball at time } t \text{ is placed in bin of size } i), \quad (4)$$

with the convention $p_{0,t} = \bar{p}$. Therefore, we can find an expression for f_i by finding an expression for $p_{i,t}$ in the RG model and then following a similar line of reasoning as in [6]. Note that (4) is equal to

$$p_{i,t} := \mathbb{P}(\text{ball at time } t \text{ increases a bins size to } i+1).$$

Rewriting this equation to the RG model it holds that,

$$p_{i,t} = \mathbb{P}(\text{arrival at time } t \text{ results in a component of size } i+1).$$

Then, let $T_x \rightarrow |C_n| = i+1$ denote a T_x arrival to component C_n augmenting its size to $i+1$ and let $T_3 + C_o$ denote that a T_3 arrival connects to component C_o . We find that for $i \geq 2$

$$\begin{aligned} p_{i,t} &= \mathbb{P}(T_2 \rightarrow |C_n| = i+1) \cdot \mathbb{P}(T_2 \text{ arrival}) + \mathbb{P}(T_3 \rightarrow |C_n| = i+1) \cdot \mathbb{P}(T_3 \text{ arrival}), \\ &= \mathbb{P}(T_2 \rightarrow |C_n| = i+1) \cdot \mathbb{P}(T_2 \text{ arrival}), \\ &\quad + \sum_{k=1}^i \mathbb{P}(T_3 \rightarrow |C_n| = i+1 \mid T_3 + C_o, |C_o| = k) \cdot \mathbb{P}(T_3 \text{ arrival}) \cdot \mathbb{P}(T_3 + C_o \mid |C_o| = k), \\ &= \frac{f_{i,t} \cdot i}{\sum_{j=1}^{\infty} f_{j,t} \cdot j} \cdot \frac{p}{\lambda+1} + \sum_{k=1}^i \frac{f_{k,t} \cdot k}{\sum_{j=1}^{\infty} f_{j,t} \cdot j} \cdot \frac{1-p}{\lambda+1} \cdot \frac{2 \cdot k \cdot (i-k+1)}{|V_t|^2 - |V_t|}, \\ &= \frac{f_{i,t} \cdot i}{\sum_{j=1}^{\infty} f_{j,t} \cdot j} \cdot \frac{p}{\lambda+1} + \sum_{k=1}^i \frac{f_{k,t} \cdot k}{\sum_{j=1}^{\infty} f_{j,t} \cdot j} \cdot \frac{1-p}{\lambda+1} \cdot \frac{2 \cdot k \cdot (i-k+1)}{\left(\sum_{j=1}^{\infty} f_{j,t} \cdot j\right)^2 - \sum_{j=1}^{\infty} f_{j,t} \cdot j}, \end{aligned}$$

with the convention that $p_{0,t} = \mathbb{P}(T_1) = \frac{\lambda}{\lambda+1}$.

Then, let $f_{i,t} \rightarrow f_i$ a.s. as $t \rightarrow \infty$. Since the RG model can be mapped to the Polya process with parameter $\gamma = 1$ by Lemma 1, the limit $\sum_{j=1}^{\infty} f_j \cdot j$ exists and is equal the average bin size, which we will denote by \bar{C} . Thus, the aforementioned assumptions also hold for the RG model. Using this and defining $c = \frac{p_{i-1} - p_i}{f_i}$, we find

$$\begin{aligned}
c \cdot f_i &= p_{i-1} - p_i, \\
&= \frac{f_{i-1} \cdot (i-1)}{\bar{C}} \cdot \frac{p}{\lambda+1} + \sum_{k=1}^{i-1} \frac{f_k \cdot k}{\bar{C}} \cdot \frac{1-p}{\lambda+1} \cdot \frac{2 \cdot k \cdot (i-k)}{\bar{C} \cdot (\bar{C}-1)}, \\
&\quad - \left(\frac{f_i \cdot i}{\bar{C}} \cdot \frac{p}{\lambda+1} + \sum_{k=1}^i \frac{f_k \cdot k}{\bar{C}} \cdot \frac{1-p}{\lambda+1} \cdot \frac{2 \cdot k \cdot (i-k+1)}{\bar{C} \cdot (\bar{C}-1)} \right), \\
&= \frac{p}{\lambda+1} \cdot \frac{f_{i-1} \cdot (i-1) - f_i \cdot i}{\bar{C}} + \frac{1-p}{\lambda+1} \cdot \left(\sum_{k=1}^{i-1} \frac{f_k \cdot k}{\bar{C}} \cdot \frac{2 \cdot k \cdot (i-k)}{\bar{C} \cdot (\bar{C}-1)} - \sum_{k=1}^i \frac{f_k \cdot k}{\bar{C}} \cdot \frac{2 \cdot k \cdot (i-k+1)}{\bar{C} \cdot (\bar{C}-1)} \right), \\
&= \frac{p}{\lambda+1} \cdot \frac{f_{i-1} \cdot (i-1) - f_i \cdot i}{\bar{C}} - \frac{1-p}{\lambda+1} \cdot \left(\frac{f_i \cdot i}{\bar{C}} \cdot \frac{2 \cdot i}{\bar{C} \cdot (\bar{C}-1)} + \sum_{k=1}^{i-1} \frac{f_k \cdot k}{\bar{C}} \cdot \frac{2 \cdot k}{\bar{C} \cdot (\bar{C}-1)} \right), \\
&\leq \frac{p}{\lambda+1} \cdot \frac{f_{i-1} \cdot (i-1) - f_i \cdot i}{\bar{C}} - \frac{1-p}{\lambda+1} \cdot \frac{f_i \cdot i}{\bar{C}} \cdot \frac{2 \cdot i}{\bar{C} \cdot (\bar{C}-1)}, \\
&= \frac{p \cdot f_{i-1} \cdot (i-1) + \left[\left(\frac{2 \cdot i}{\bar{C} \cdot (\bar{C}-1)} - 1 \right) \cdot p - \frac{2 \cdot i}{\bar{C} \cdot (\bar{C}-1)} \right] \cdot f_i \cdot i}{(\lambda+1) \cdot \bar{C}}.
\end{aligned}$$

And therefore, for $i \geq 2$ it holds that

$$f_i \leq \frac{p \cdot (i-1)}{(\lambda+1) \cdot c \cdot \bar{C} - \left[\left(\frac{2 \cdot i}{\bar{C} \cdot (\bar{C}-1)} - 1 \right) \cdot p - \frac{2 \cdot i}{\bar{C} \cdot (\bar{C}-1)} \right] \cdot i} \cdot f_{i-1}.$$

Using this expression and defining $f_i \propto g(i)$ as $f_i = c(1 + o(1))g(i)$, we find that

$$\begin{aligned}
f_i &\leq f_1 \cdot \prod_{j=2}^i \frac{p \cdot (j-1)}{(\lambda+1) \cdot c \cdot \bar{C} - \left[\left(\frac{2 \cdot j}{\bar{C} \cdot (\bar{C}-1)} - 1 \right) \cdot p - \frac{2 \cdot j}{\bar{C} \cdot (\bar{C}-1)} \right] \cdot i}, \\
&\propto \prod_{j=2}^i \frac{p \cdot (j-1)}{\lambda+1 + p \cdot j} = \prod_{j=2}^i \frac{j-1}{j + \frac{\lambda+1}{p}} \propto \frac{\Gamma(i)}{\Gamma\left(i+1 + \frac{\lambda+1}{p}\right)} \propto i^{-(1+\frac{\lambda+1}{p})}. \quad (5)
\end{aligned}$$

which indicates power-law behaviour.

Validation of results

To validate these results, we ran multiple simulations using our model and plotted the probability density function (pdf) for each of these runs. We compare

these simulations to the Yule distribution, with parameters α and x_{\min} . Since we are analysing the distribution of component sizes, it follows that $x_{\min} = 1$. Then, from Equation 5, we find that $\alpha = \frac{\lambda+1}{p} + 1$. Let $\Gamma(\cdot)$ denote the Gamma function, by [7], the pdf of this distribution is as follows,

$$f(x) = \left(\frac{\lambda+1}{p}\right) \frac{\Gamma\left(\frac{\lambda+1}{p} + 1\right)}{\Gamma(1)} \frac{\Gamma(x)}{\Gamma\left(x + \frac{\lambda+1}{p} + 1\right)}. \quad (6)$$

In Figure 2 we depict the results for several values of p of the simulations with $t = 100,000$, $\lambda = \frac{1}{3}$ and $q = 0.9$ as values for the other parameters of the RG model. Also depicted in Figure 2 as black squares are the values of the pdf of the Yule distribution, shown in Equation 6.

In Figure 2 we see that for every run, there is one really large component. In [24], we named these components the Largest Connected Component (LCC) and we mention the fraction of nodes in the LCC in the legend of Figure 2. Note that all the values for the other component sizes are slightly below the Yule distribution. This fact supports our claim that the component size distribution shows power-law behaviour. These values are slightly below the Yule distribution, since Equation 5 is an upper bound for f_i .

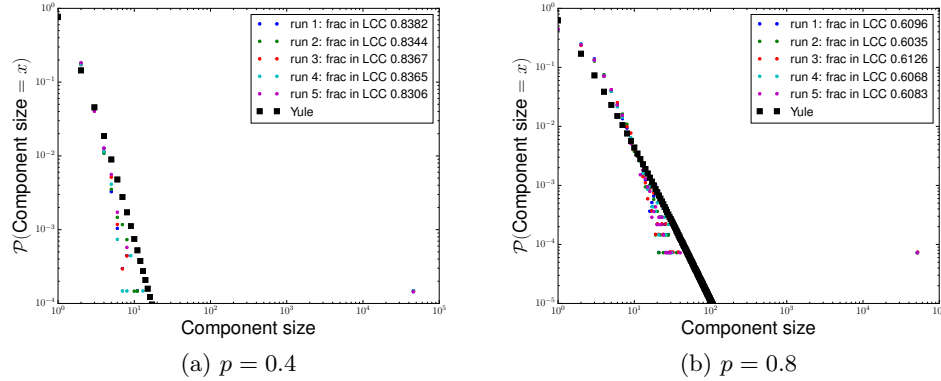


Fig. 2. Plots of the pdf of the component size distribution. For these simulations we used $t = 100,000$, $\lambda = \frac{1}{3}$ and $q = 0.9$.

5 Conclusion and Discussion

In this paper, in which we have extended our previous work on the RG model, we derived the limiting behaviour of the component sizes. Through a mapping

of the RG model to the Polya process, we found that the distribution of component sizes shows power-law behaviour. Note that Corrolary 3 is identical to Theorem 4 when we fill in $p = 1$. Thus, interestingly enough, the possibility of merging components does not seem to affect this limiting behaviour greatly. The validation in this paper is based on simulated results, thus the model has currently not been tested to *Twitter* datasets.

Moreover, the model used in this paper can also easily be extended to a less general setting. For instance, the superstar parameter q is assumed to be equal for every message tree. This can be easily extended to an individual superstar parameter per message tree q_i . This addition does not change the result of the analysis shown in this paper and therefore is not included here. Another aspect that could be taken into account in future work, is the weight of an edge. Given that there are multiple retweets between u and v , this can be taken into account by adding a weight to every edge. A last extension that could prove to be interesting, is to use time-varying parameters. For simplicity, this has not been taken into account.

We plan to explore these aspects in our future research.

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